

Coevolution of Behavior and Networks in Games of Cooperation and Coordination

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Abstract

Social structure is both a consequence and a determinant of human behavior. In order to shed light on the problem of the emergence and maintenance of social order, one of the central underlying quests in social science, we need to understand how behavior and structure coevolve. This paper discusses how this coevolution process can be modeled with cooperation and coordination games on dynamic networks. I review models from recent analytical, simulation-based, and experimental studies by reconstructing them within a general formal framework. The analysis reveals that in theoretical studies, the relative speed of network update (how often actors reconsider their links compared to their action choice) is one of the factors with the biggest impact on macro-outcomes such as efficiency, hierarchical organization and inequality. However, this effect is conditional on one assumption that is common to all existing models, namely, that players employ the same action against all of their partners. I argue that future research should relate models to applications and experimental tests more adequately, which often implies allowing for discriminatory action.

1. Introduction

The problem of social order is undoubtedly one of the most central underlying quests in social science. The idea of order concerns two conceptually distinct aspects: social structure (forms of relating) and human action (ways of behaving). Thus, two of the most widely studied subject areas by social scientists are the formation, evolution, and reproduction of patterns of interaction (e.g. organizations) and the emergence and maintenance of cooperation (e.g. collective action) and coordination (norms, conventions, etc.).

The two aspects of social order, however, are not independent: social structure is both a consequence and a determinant of human behavior (Coleman, 1990). Hence, in order to truly

grasp how social order emerges and persists, we need to understand how behavior and structure coevolve. In this paper, I address the question of how this coevolution process can be scientifically modeled. My goal is twofold: first, to critically reconstruct coevolution models from recent analytical studies, simulations and experiments and second, to summarize results, point biases, discover gaps and indicate directions for further research.

The analysis focuses on a conceptualization of order as a stable network of relations that emerges from the dynamics of incentive-guided individual behavior. Particularly, I concentrate on game-theoretic approaches to network formation and put aside random-graph and stochastic-process models (e.g. Watts and Strogatz, 1998; Barabasi and Albert, 1999; Robins et al., 2005). Additionally, I only cover game-theoretic models from the social sciences and neglect comparable developments by evolutionary biologists, physicists, computer scientists and statisticians (e.g. Skyrms and Pemantle, 2000; Biely et al., 2007; Snijders et al., 2007), as they differ in methodological approach and substantive focus. Furthermore, I concentrate on how networks coevolve with individual behavior as actors search for partners to interact with. I do not consider the other approach to network formation explored in the social-scientific game-theoretic literature: networks evolving due to individuals striving for particular network positions (e.g. Buskens and van de Rijt, 2008; Burger and Buskens, 2009; for a theoretical overview, see Jackson, 2005 and Jackson, 2008; for an overview of recent experiments, see Kosfeld, 2003). I also limit the analysis to three different games of coordination and cooperation: the Coordination Game, Prisoner's Dilemma and Hawk-Dove Game. I do not explore games of exchange, which have been extensively studied out of the context of two-player two-action non-cooperative games (Willer, 1999; for studies of dynamic exchange networks, see Willer and Willer, 2000; Pujol et al., 2005; Dogan et al., 2009).

The paper is structured as follows. I first delineate a general formal framework that reconstructs all reviewed models as a social game consisting of a network game and an underlying non-cooperative 2x2 game. I then discuss the specifics of the stages of modeling the social game: the link formation stage (the definition of links and the cost function for links), the game playing stage (the strategies and payoffs in the three games), the sequence of choices and the "solution" (the equilibrium and stability concepts). Next, I combine analytical solutions, simulation results and experimental findings to relate the model parameters to macro-outcomes such as network structure and global behavior, as well as particular network-behavior patterns. Last, I elaborate on the common assumption of players using the same action against all partners and provide suggestions how my critique could be addressed in future research.

2. Formal framework for social games

As a new research line that has emerged and shaped up only in the past decade, the study of the coevolution of behavior and networks has been undergoing a rapid but largely disorganized development. Currently, there is a variety of analytical studies, simulations and experiments, all based on models with different assumptions and variables. In order to facilitate the comparison of the various approaches and their results, this section delineates a general formal framework that fits the game-theoretic coevolution models found in the literature. The base model has two components: a network game (defining an interaction network) and an underlying game (a coordination/cooperation game played by connected actors). The combined strategies of the two games define the strategy of what, in line with an emerging convention, I call a social game.

Formally, let $N = \{1, 2, \dots, n\}$ be a finite set of players, where we assume that the number of players $n \geq 3$, as two-person networks are trivial. Let $g_i = (g_{i1}, \dots, g_{in})$ be the vector of link decisions of player i , where $g_{ij} = \{0, 1\}$ and $i, j \in N$, $i \neq j$; by convention, $g_{ii} = 0$ for all i , that is, ties are non-reflexive. The set of all possible individual network decisions is $G \subseteq \{0, 1\}^{n(n-1)}$. Since the literature uses two different approaches to forming links (see next section), it will also be useful to define a symmetrized version of g_i , denoted by \bar{g}_i and indicating the established links for player i . Thus, $\bar{G} \subseteq \{0, 1\}^{n(n-1)/2}$ defines a graph with its nodes representing the players and its edges – the established symmetric links. Furthermore, let $N_i \equiv \{j \in N : g_{ij} = 1\}$ denote the subset of players to whom i has proposed a link and $\bar{N}_i \equiv \{j \in N : \bar{g}_{ij} = 1\}$ – the players i interacts with. We define $n_i = |N_i|$ as the cardinality of N_i , i.e. the number of actors to whom i has proposed a connection and respectively, $\bar{n}_i = |\bar{N}_i|$ as the number of i 's interaction partners. Lastly, since both initiating and preserving social interactions usually involve some time and effort, we allow links to be costly: we define a cost function $\phi(n_i, \bar{n}_i)$ for proposing n_i links and/or participating in \bar{n}_i links.

It is generally assumed that players i and j play the underlying game if they are connected in \bar{G} .¹ We define the underlying game Γ to be a two-player two-action symmetric

¹ Exceptions are Hojman and Szeidl (2006) and Mengel and Fosco (2007) who consider interaction with both direct and indirect neighbors. Arguably, the assumption that an actor can play a game with indirect neighbors is theoretically problematic: it is analogous to modeling an N -person game while ignoring communication costs and obscuring the true interaction structure.

game in normal form with a common action set $A = \{X, Y\}$ and a payoff function π , where $A \times A \rightarrow \mathbb{R}$ and player i 's action profile is a_i .

Finally, we model the strategic situation of the player in the social game Γ^S as a combination of her strategies in the network and the underlying game: $s_i = (g_i, a_i)$, where $g_i \in G$ and $a_i \in \{X, Y\}$ in Γ . The player's total payoff consists of the payoff she obtains from playing the underlying game with her neighbors minus the costs of her links. In formal terms, in the social game Γ^S , given the strategies of the other players $s_{-i} = (s_1, \dots, s_{i-1}, s_{i+1}, \dots, s_n)$, the payoff of player i from playing some strategy $s_i = (g_i, a_i)$ is $\Pi_i(s_i, s_{-i}) = \sum_{j \in \bar{N}_i} \pi(a_i, a_j) - \phi(n_i, \bar{n}_i)$.

3. Stages in modeling social games

The two components of the social game imply four distinct aspects of the coevolution process that need to be modeled: how links are formed, how the underlying game is played, what the sequence of link and action decisions is and when equilibrium/stability is reached. In what follows, I review the approaches and concepts available at each of these four modeling stages. The goal is to both flesh out the general framework presented above and systematically reconstruct the models in the coevolution literature in terms of assumptions and variables (see Table 2), which will enable the subsequent analysis of their results.

3.1. Link formation

3.1.1. Definition of links

Coevolution models borrow from the strategic-networking literature two different approaches to defining links: unilateral link formation (Bala and Goyal, 2000) and bilateral link formation (Jackson and Wolinsky, 1996). The two approaches do not necessarily lead to the same predictions. They also apply to different social situations and may involve the use of different equilibrium/stability concepts.

In unilateral link formation, a link ij is formed whenever $\max\{g_{ij}, g_{ji}\} = 1$, that is, if at least one of the players wants to establish the link. Thus, i 's interaction neighborhood is defined by $\bar{g}_{ij} = \max\{g_{ij}, g_{ji}\}$ and hence, the players to whom i has proposed a link are only

a subset of the players with whom i interacts: $N_i \subseteq \bar{N}_i$. This definition of links is suitable to applications where all types of interactions are profitable (as a rational player would always accept a proposed link because she would lose payoff otherwise) and is usually used to model situations where links are costly only for one of the partners. The assumption of unilateral link formation is also methodologically advantageous: it simplifies the analysis because it allows for the use of best-response strategy at the individual level and (strict) Nash equilibrium at the collective level (see section 3.4.1 for definitions).

In bilateral link formation, a link ij is formed whenever $\min\{g_{ij}, g_{ji}\} = 1$, or if and only if both players want to have the link. Correspondingly, i 's interaction neighborhood is $\bar{g}_{ij} = \min\{g_{ij}, g_{ji}\}$ and the players in it are only a subset of those to whom i would like to link: $\bar{N}_i \subseteq N_i$. This approach to defining links implies that although each player can unilaterally sever any of her links, the mutual consent of both players is required in order for a connection to exist. Initially developed in the context of cooperative games, bilateral link formation has required the definition of additional stability concepts: pairwise stability (Jackson and Wolinsky, 1996), strong pairwise stability (Gilles et al., 2006) and unilateral stability (Buskens and Van de Rijt, 2008). However, the assumption that the network game is of the cooperative type is not needed if the link decisions are considered to be independent. Furthermore, such an assumption is unjustified in social games, given that the underlying game is assumed to be non-cooperative. Hence, coevolution models with bilateral link formation have employed both the Nash equilibrium and stability concepts based on pairwise stability (discussed in section 3.4).

3.1.2. Cost functions for links

The relative cost of links (in relation to the payoffs of the underlying game) changes the incentives at the individual level and carries major implications for both the network structure and the global behavior in the stable states. Link costs can represent the information costs in finding a partner and establishing a contact (costs for initiating a link), the time and effort involved in communication (costs for maintaining a link), as well as the disutility from terminating a relationship (costs for breaking a link). Additionally, costs can be either shared, or incurred by one party only (usually, the initiator of the link). The major difference between the two is that one-sided link costs bring forth an externality issue: a player may benefit from a connection even if she does not pay for it.

Despite the numerous options, all existing models in the literature assume costs for link maintenance. In addition, models with unilateral link formation always use one-sided costs for the link-initiator (e.g. Berninghaus and Vogt, 2003; Bramoullé et al., 2004; Goyal and Vega-Redondo, 2005), while models with bilateral link formation assume that both partners pay for the link (e.g. Droste et al., 2000; Buskens et al., 2008).

Apart from deciding who pays the link costs, it is also important to define what the costs look like. For example, links can have the same cost regardless of the number of partners one already has ($\phi = kn_i$ if costs are incurred by the link initiator and $\phi = k\bar{n}_i$ if costs are shared, $k > 0$). Special cases of a linear cost function for links are when costs are zero (equivalent to a shift in payoffs $(\pi - k)$ where $k < \min(\pi)$) or when one is compensated for any ties not formed ($\phi = -k(n - \bar{n}_i)$). Although the latter case is typically used in experimental settings to guarantee a minimum compensation to all participants, it could also be applied to study exclusion (Riedl and Ule, 2002).

The assumption of constant costs implies that one can maintain an unlimited number of partnerships, as long as they are profitable. However, a more realistic assumption would be that after a certain number of relationships, it becomes unfeasible to maintain additional ones. This idea has been modeled in two different ways (illustrated here for two-sided linking): by imposing a threshold m , where $n > m > 1$ and such that $\phi(\bar{n}_i) > \max(\pi)$ for all $\bar{n}_i > m$ (Jackson and Watts, 2002), or by specifying increasing marginal (i.e. convex) costs $\phi = k\bar{n}_i + l\bar{n}_i^2$ with $k, l > 0$ (Jackson and Watts, 2002; Buskens et al., 2008) or more generally, $\phi = k\bar{n}_i^\lambda$ with $k > 0$ and $\lambda > 1$ (Hanaki et al., 2007).

3.2. *Playing the game*

After the interaction network is established, actors play the same underlying game with each of their partners. All reviewed social-game models assume that players employ the same action in all of the bilateral games they are simultaneously engaged in. I elaborate on how this assumption relates to applications and affects results in the final section. In what follows, I briefly present the applications and the equilibrium solutions of the three games most commonly used to model problems of cooperation and coordination: Coordination Game,²

² In order to simplify the analysis, I focus only on pure coordination games. Another game which is commonly used to represent problems of coordination is Stag-Hunt. Within the coevolution framework, there are two prominent studies of this game: a simulation-based analysis by Skyrms and Pemantle (2000), seminal in the modeling of endogenous dynamic network structure, and an experiment by Corbae and Duffy (2008). Skyrms and Pemantle assume memory-based learning and use reinforcement-based dynamics, while Corbae and Duffy

Prisoner's Dilemma and Hawk-Dove Game. Table 1 gives the payoff matrices, the strategies and the outcomes of the one-shot symmetric two-player two-action games in normal form.

TABLE 1
GAMES OF COOPERATION AND COORDINATION *

	Coordination Game			Prisoner's Dilemma			Hawk-Dove Game		
Matrix		X	Y		X	Y		X	Y
	X	b,b	e,d	X	c,c	e,b	X	c,c	d,b
	Y	d,e	c,c	Y	b,e	d,d	Y	b,d	e,e
Pure Strategies		X Y			X: cooperate Y: defect			X: dove Y: hawk	
Pure-Strategy Nash Equilibria		(X,X) (Y,Y)			(Y,Y) dominant			(X,Y) (Y,X)	
Mixed-Strategy Nash Equilibria		$\rho_x = \frac{c-e}{b-d+c-e}$						$\rho_x = \frac{d-e}{b-c+d-e}$	
Pareto Optimal Outcomes		(X,X)			(X,X) (X,Y) (Y,X)			(X,X) (X,Y) (Y,X)	
Maximum Welfare		(X,X)			(X,X) if $2c > b + e$			(X,X) if $2c \geq b + d$ (X,Y) if $2c \leq b + d$ (Y,X)	
Additional Constraints		If $c + d > b + e$: (X,X) payoff-dominant (Y,Y) risk-dominant							

* $b > c > d > e$

3.2.1. Coordination Game

In the Coordination Game, the two players realize highest gains only when they choose the same action. The game has two Nash equilibria in pure strategies and one in mixed strategies. The pure-strategy equilibria always Pareto-dominate the mixed-strategy one. In order to further problematize the welfare implications, it is often assumed that the two pure-strategy equilibria are Pareto-ranked themselves: in Table 1, {X,X} is the payoff-dominant equilibrium and {Y,Y} – the inefficient. The game also has a risk-dominant equilibrium in the sense that each player chooses the strategy that is a best response to the other player mixing with equal probability. When the efficient action is the more risky one, reaching a socially optimal outcome becomes even more problematic and the coordination problem turns into a

model the social game as a game with incomplete information and employ perfect Bayesian equilibrium solutions.

social dilemma. Thus, although most commonly associated with the evolution of conventions (such as languages, currencies, product standards, etc.), the game has been used to model the dilemma of collective action as well (Heckathorn, 1996).

3.2.2. Prisoner's Dilemma

The Prisoner's Dilemma is an extreme example of a situation in which individually-rational decisions lead to suboptimal outcomes at the collective level. Players have a choice between two actions, "cooperate" and "defect", but whichever action one of the players chooses, the other one is always better off if she defects. Thus, although cooperation yields the maximum mutual benefit, defection is a dominant strategy and the Nash equilibrium predicts that both players defect.

3.2.3. Hawk-Dove Game

The Hawk-Dove Game (also known as Chicken Game) is a game that combines both the coordination and the cooperation problem. Maynard Smith (1973) famously introduced the game in the field of evolutionary biology to model individual selection within species and used it to develop the concept of evolutionary stable strategies. He suggested an interpretation in terms of competition for territory or resources. If the two players cooperate (Doves), they divide the resource equally among themselves; if one individual acts aggressively (Hawk), she either receives most of the resource if her opponent retreats (Dove) or they both end up with a minimum payoff due to an ensuing fight (Hawk-Hawk).

In the Hawk-Dove Game, one's best response is to choose an action unlike the action of one's partner. Hence, the only Nash equilibria in pure strategies are the asymmetric strategy configurations (Hawk, Dove) and (Dove, Hawk); the only symmetric equilibrium is the mixed-strategy equilibrium. If $2c > b + d$, reciprocated Dove-behavior yields the highest combined benefit. However, mutual cooperation is never an equilibrium and this presents a social dilemma. If the opposite is true ($2c < b + d$), dissimilar actions yield highest total benefit. This presents a coordination problem, aggravated by the fact that both players' failure to yield (reciprocated Hawk-behavior) is disastrous. Because of these properties, the game has been used to represent brinkmanship and appeasement (Rapoport and Chammah, 1966), conflict (Bornstein et al., 1997) and collective action (Heckathorn, 1996).

3.3. Sequence of choices

After determining how players form links and choose actions, it is important to decide in what order they take these two decisions. One approach is to model the partner and action choices as simultaneous: actors choose the type of change (a link, behavior, or a combination of the two) that provides the highest payoff. This approach applies to both static models (assuming a one-shot social game Γ^S) and dynamic models (using a repeated Γ^S). Assuming simultaneous link and action changes is particularly suitable for situations with well-informed players interacting in small networks. In fact, such players should also be capable of looking one step further to the implications of their decisions for the future evolution of the play. Thus, the simultaneous consideration of network and action decisions allows for modeling forward-looking behavior in social games (see Berninghaus et al., 2008).

The other possible approach is to assume that link and action choices are taken in a certain sequence. The assumption represents players in larger networks who have limited and local information. The approach requires a dynamic framework of discrete time and hence, it is most commonly used in simulations. At every period of the dynamic social game, three events happen:

- a random pair of players is chosen with a probability $p_{ij} > 0$ and the two actors decide whether they want to add or remove a link between themselves;
- a random player is chosen with a probability $q_i > 0$ and the player decides whether to change her behavior;
- all players play the game and obtain their payoffs.

These events may occur in any order at the modeler's discretion. The most noteworthy implication from assuming separate link and action choices, however, is that p_{ij} and q_i can be varied independently, thus modifying the "speed" of network updates relative to the frequency of action updates. This allows us to model the volatility of social relations (related to, for example, geographical mobility) in a simulated society.

3.4. Equilibrium/Stability

To define the theoretical solutions to the social game, static models usually employ the Nash equilibrium or refinements and extensions of it. Dynamic models, regardless of whether they assume simultaneous or sequential link and action choices, typically rely on stability concepts.

3.4.1. Equilibrium concepts

Best response is a strategy that produces the most favorable outcome for a player, taking other players' strategies as given. This concept is central to the definition of Nash equilibrium: in a Nash equilibrium, each player has selected the best response to the other players' strategies. Thus, no player has an incentive to change strategy. Formally, a strategy profile $s^* = (s_1^*, \dots, s_n^*)$ is a Nash equilibrium of Γ^S if $\forall i \in N$ and $\forall s_i \in S_i$, $\Pi_i(s_i^*, s_{-i}^*) \geq \Pi_i(s_i, s_{-i}^*)$. The solutions predicted by this concept can be narrowed down by changing the inequality in the definition to a strict one. This refinement is known as strict Nash equilibrium and in it, every player gets a strictly higher payoff with her current strategy than she would with another one available.

Extensions of the (strict) Nash equilibrium concept can come from different behavioral assumptions. For example, in order to better explain their experimental results, Berninghaus et al. (2008) replace the common assumption of myopic behavior³ with forward-looking belief-formation. They extend the static-game Nash equilibrium to "one-step-ahead" stability (see Berninghaus et al., 2008 for definition). Although this concept implies a repeated game, dynamic models usually employ an entirely different approach to defining stability.

3.4.2. Stability concepts

In dynamic models, the link and action update mechanisms define an evolutionary process. Starting from an initial state with random distribution of links and behavior, the feedback between partner and action choices results in a dynamic evolution that converges on any of a finite number of possible stable states, i.e. states in which both the neighborhood and the behavior of each agent do not change over time. Formally, the stable states in a dynamic social game are states in which:

- no player wants to sever a link or change action, and
- no player wants to establish a link (if the link formation is unilateral) or no pair of players wants to establish a mutual link (if the link formation is bilateral).⁴

A dynamic system can have a large number of stable states. Which one is more likely depends on the system's initial state. Hence, the probability of a particular outcome can be statistically estimated over a sample of initial configurations.

³ In the context of coevolution games, myopic actors optimize under the assumption that the network and the behavior from the previous period remain the same in the current period.

⁴ The stability concept for social games with bilateral link formation corresponds to Jackson and Wolinsky's (1996) pairwise stability for networks where only links can be changed.

One common way to refine the set of predicted stable states is to use the concept of stochastic stability.⁵ Stochastic models assume trembles in the players' decisions due to external perturbations, limitations in the player's calculative ability, experimentation or simply mistakes in executing the best-response strategy.⁶ Models with sequential link and action decisions allow the errors for the two to be independent: the action is correctly implemented with probability $1-\varepsilon$, while the link – with probability $1-\tau$, where $1 > \varepsilon, \tau > 0$.⁷ Models with simultaneous link and action update assume that the player implements her best-response strategy in the social game with probability $1-\eta$, where $1 > \eta > 0$. A state is stochastically stable if the probability that the system will be in that state in the long run is bounded away from zero as the error probabilities become infinitely small (Foster and Young, 1990). In practical terms, the higher the number of mistakes needed for the system to leave a stable state, the more stochastically stable that state is. Thus, the introduction of mutation rates in the dynamic process leads to more precise long-run predictions.

4. Social games of coordination and cooperation

In the previous two sections, after outlining a common framework for social games, I reviewed how models differ in link definition, link costs, payoffs of the underlying game, sequence of choices, and equilibrium/stability concepts. In each of the existing models, researchers fix some of these parameters as assumptions and explore the effects from varying the rest. Table 2 summarizes the assumptions, variables, and results of the models in the literature.

In this section, I link the model parameters to the macro-outcomes. I concentrate on the network, the global behavior and the specific network-behavior patterns in the stable configurations of the social games. I discuss the outcomes in terms of efficiency, payoff distribution, role differentiation, hierarchical structuring, network density, segregation, and polarization – all aspects that in one way or another relate to the three major themes of sociology, namely, inequality, rationalization, and cohesion.

⁵ For models with bilateral link formation, another way to narrow down predictions is to combine the requirement of stability in behavior with one of the refinements of pairwise stability: strong pairwise stability (Gilles et al., 2006) or unilateral stability (Buskens and Van de Rijt, 2008).

⁶ Technically speaking, the random mechanism for updating links and actions introduces a stochastic element to all dynamic models.

⁷ Among the reviewed models, only Hojman and Szeidl (2006) model the two error probabilities as related: the probability for making a mistake in the linking decision is ε^τ . This implies that the link and action mistake probabilities approach zero at different rates, which changes the results significantly (Bergin and Lipman, 1996).

TABLE 2
COEVOLUTION OF BEHAVIOR AND STRUCTURE IN GAMES OF COOPERATION AND COORDINATION ^a

Study	Type ^c	\bar{g}_{ij}	ϕ	Sequence of Choices	Equilibrium/ Stability	Behavioral Assumption	Variables	Results ^e	Network and Global Behavior ^f	
Coordination										
Droste et al., 2000	An	$\min\{g_{ij}, g_{ji}\}$	$\sum_{j \in N_i} k_{ij}$ ^d	$g_{ij} \rightarrow a_i, a_j \rightarrow s$	deterministic	myopic BR	$b/\gamma \geq k_{ij}$	X--X	↑efficiency	
							$c/\gamma \geq k_{ij}$	Y--Y		
								X-X Y-Y	segregation	
Jackson & Watts, 2002	An	$\min\{g_{ij}, g_{ji}\}$	$k\bar{n}_i$	$g_{ij} \rightarrow a_k \rightarrow s$	stochastic	myopic BR	$\tau, \varepsilon > 0$	$b > k > d,$ $\rho_Y > 1/(n-1)$	X-X	
								$b > k > c,$ $\rho_Y \leq 2/(n-1)$	Y Y	
								$c > k$	Y-Y	
								$b > k > d$	X--X	
								$c > k > d$	Y--Y	
								$d > k$	Y-m-Y	
								$k\bar{n}_i,$ $\bar{n}_i \leq m$		
Berninghaus & Vogt, 2003	An	$\max\{g_{ij}, g_{ji}\}$	kn_i	static	Nash	myopic BR	$b > k$	X-X	↑efficiency	
							$b > k > c$	Y Y		
							$c > k$	Y-Y	segregation	
							$c > k > d$	X-X Y-Y		
							$d > k > e$	X-X → Y-Y		
Goyal & Vega-Redondo, 2005	An	$\max\{g_{ij}, g_{ji}\}$	kn_i	simultaneous	stochastic	myopic BR	$\eta > 0$	$c > k^* > e$	X-X	↑efficiency
								$k > k^*$	Y-Y	↓efficiency
								$k^* > k$		
Hojman & Szeidl, 2006 ^b	An	$\max\{g_{ij}, g_{ji}\}$	kn_i	simultaneous	stochastic	myopic BR	$\varepsilon^r, \varepsilon > 0$	$k \approx 0$	Y-Y wheel	
								$k \approx 0, \sum_{j \in N_i} g_{ij} < 2$	Y-Y wheel	
									X-X wheel	
Buskens et al., 2008	Sim	$\min\{g_{ij}, g_{ji}\}$	$k\bar{n}_i + l\bar{n}_i^2$	$s \rightarrow g_i a_i$ $s \rightarrow g_{ij} \rightarrow a_k$ $s \rightarrow [n/2] g_{ij} \rightarrow a_k$	deterministic	myopic BR	$[n/2] g_{ij} \rightarrow a_k$	segregation	X--X	↑polarization
							density	Y--Y	↓ n_X ↑polarization	
							n_X	X--X Y--Y	↓polarization	
							density. n_X			
							$n \downarrow$		↑ n_X	
Corten & Buskens, 2008	Exp	$\min\{g_{ij}, g_{ji}\}$	$k\bar{n}_i + l\bar{n}_i^2$	$n g_{ij} \rightarrow a \rightarrow s$	deterministic	myopic BR	n_X	X-m-X	↑efficiency	
							density	X-X Y-Y	↓efficiency	
							Y--Y			

Prisoner's Dilemma									
Riedl & Ule, 2002	Exp	$\min\{g_{ij}, g_{ji}\}$	$-k(n - \bar{n}_i)$	simultaneous	Nash, Subgame Perfect	exclusion			$n_X > 0$
						global info	$e > k$	X--X--Y--Y	\uparrow density
						local info	$c > k > d$	X--X - Y--Y	$\uparrow n_X \uparrow$ density
Eguíluz et al., 2005	Sim	$\max\{g_{ij}, g_{ji}\}$	0	$s \rightarrow a \rightarrow g_{unsatisfied Y}$	deterministic	imitation			
						random search	$p_{ij} = 0, b \uparrow$	Y--X--X--Y	$\downarrow n_X$
						local search	$p_{ij} \gg 0, b \uparrow$	Y--Y	\uparrow inequality \uparrow hierarchy
Hanaki et al., 2007	Sim	$\min\{g_{ij}, g_{ji}\}$	$k\bar{n}_i^\lambda$	$s \rightarrow \alpha a_i \rightarrow \beta g_{jk}$	stochastic $\tau = \varepsilon > 0$	imitation	$n \uparrow$		$\uparrow n_X$
						local + random search	$k, \lambda \uparrow$		$\uparrow n_X \downarrow$ density
							$k, \lambda \uparrow \beta / \alpha \uparrow$		$\uparrow n_X$
Mengel & Fosco, 2007 ^b	Sim	$\min\{g_{ij}, g_{ji}\}$	0, $\bar{n}_i \leq m$	$a_i \rightarrow \beta g_{jk} \rightarrow s$	stochastic $\tau, \varepsilon > 0$	imitation	$r = 2$	X--X--Y--Y	
						local search (radius r)	$r \geq 2$	X--X Y--Y	segregation
							$r > 2, \beta \uparrow$	Y--Y	$\uparrow n_X$
							$r \uparrow$	\downarrow clustering \downarrow avg. distance	
Hawk-Dove									
Berninghaus & Vogt, 2003	An	$\max\{g_{ij}, g_{ji}\}$	kn_i	static	Nash	myopic BR	$b > k > c$	$\frac{X \ X}{Y \rightarrow X}$	$n \geq n_Y \geq 0 \downarrow$ density
							$c > k > d$	Y \rightarrow X-X	$n \geq n_Y \geq (n-1)\rho_Y$
Bramoullé et al., 2004							$d > k > e$	Y-X-X	$\downarrow n_Y$
							$e > k$	Y-Y-X-X	$(n-1)\rho_Y + 1 \geq n_Y \geq (n-1)\rho_Y$ \uparrow density
Berninghaus et al., 2008	Exp	$\max\{g_{ij}, g_{ji}\}$	kn_i	simultaneous	one-step-ahead	forward- looking belief formation	$b > k > c$	Y \rightarrow X	$\frac{n\Delta + d}{\Delta + d} \geq n_Y \geq \frac{(n-1)\Delta}{\Delta + d}$
							$c > k > d$	Y \rightarrow X-X	$\frac{n(b-c) + d}{b-c+d} \geq n_Y \geq 0$
							$d > k > e$	Y-X-X	$\uparrow n_Y$
							$e > k$	Y-Y-X-X	$(n-1)\rho_Y + 1 \geq n_Y \geq (n-1)\rho_Y$

a. For notation used, see text and Table 1.

b. Interaction with both direct and indirect neighbors.

c. An analytical study; Sim simulation; Exp experiment

d. $k_{ij} = \min\{\gamma|j-i|, \gamma|j-i+n|\}$, where γ is the cost per distance unit (players are distributed on a circle)

e. Y--Y links between Y-players; X--X complete network between X-players; X - Y rare links between X-players and Y-players; X \rightarrow Y X-players linked to Y-players

f. n_X with some abuse of notation, the proportion/number of X-players; Δ ($b-k$)

4.1. Coordination Game

If individuals play the Coordination Game with partners selected at random from the population, the only stochastically stable equilibrium is the risk-dominant strategy profile (e.g. Young, 1993). This result is due to the fact that the risk-dominant strategy yields the highest payoff against an average population: it is a best response if the fraction of the population playing the same strategy is $\frac{b-d}{b-d+c-e}$ (the mixing probability for action Y in the mixed-strategy equilibrium), which is always less than 0.5. When players can strategically choose their partners, however, coordinating on the Pareto-efficient equilibrium is possible. What is more, different parts of the population can simultaneously coordinate on different equilibria.

One factor that increases the likelihood of Pareto-efficient coordination is the proportion of players choosing the payoff-dominant strategy in the initial stage of the social game. This effect has been obtained in simulations (Buskens et al., 2008) and confirmed in an experiment (Corten and Buskens, 2008). Analytical studies have explored another factor that facilitates efficiency: the cost of links. Coordination on the payoff-efficient equilibrium is the only possible solution in a connected network if the link costs are high enough, i.e. $b > k > c$ (Droste et al., 2000; Berninghaus and Vogt, 2003). In this case, playing the inefficient equilibrium does not pay off after subtracting the communication costs. For lower link costs, efficiency is possible but not guaranteed in deterministic and static models. Moreover, below a certain cost threshold k^* , $c > k^* > e$, efficiency is not stochastically stable (Jackson and Watts, 2002; Goyal and Vega-Redondo, 2005). The intuition behind this last result is that when link costs are low, the players are maximally connected and the situation resembles the base case with random interaction: more than half of one's partners need to randomly switch to the efficient action in order for one to do so rationally. Yet, efficiency is also not stochastically stable when interaction is restricted. For example, the model of Droste et al. (2000) localizes interactions by assuming costs based on geographical distance. In the resulting neighborhood-structured networks, the risk-dominant convention is the only stochastically stable state.

Stochastic models show that the social game of coordination converges to a state where all players behave uniformly. However, the coexistence of both equilibria is theoretically possible for low link costs in deterministic environments (Droste et al., 2000; Buskens et al., 2008) and in static models (Berninghaus and Vogt, 2003). It is also experimentally observed (Corten and Buskens, 2008).

Since miscoordination is costly, the coexistence of equilibria implies that ties will occur mainly between actors with the same behavior. The segregation of the network does not have to be complete only when link costs are one-sided and low enough (Berninghaus and Vogt, 2003). In this case, it is still profitable for X-players to sponsor connections to Y-players.

When the network is segmented into components, it is also important to know to what extent the separate groups are of similar size. Buskens et al. (2008) conceptualize this as polarization and find out that polarization is both more likely and more extreme for “faster” networks (models with link updates for $n/2$ randomly selected pairs at every period rather than a single link update). The large number of link changes causes the network to fall apart into groups with different behavior before the actors get a chance to adapt their behavior.

4.2. Prisoner’s Dilemma

If players use best response in a repeated Prisoner’s Dilemma, the Nash equilibrium predicts universal defection, as defection is the dominant strategy. Non-trivial solutions require additional behavioral assumptions such as imitation learning (emulating the action or link choices of successful players), informed partner search⁸ (using heuristic rules to find partners who are likely to cooperate), and optional participation (avoiding defectors). Imitation, however, affects cooperation in both directions: while it enables individual cooperation, it precludes full cooperation as a global outcome. Firstly, since unilateral defection is the most profitable action choice, if players imitate those with highest payoffs, defectors are likely to survive under deterministic dynamics. In addition, because a single random mistake could lead to the viral replication and spread of defection, full cooperation is not stochastically stable either.

Nevertheless, in social games, the global level of cooperation could increase under specific conditions. In their simulation study, Mengel and Fosco (2007) find that when players interact more often among each other, they begin to exclude defectors often enough to compensate for the also increased tendency to emulate them. Thus, given that their search radius is large enough, the more often players update their link choices, the higher the number of cooperators. Hanaki et al. (2007) reproduce the finding that higher relative network speed increases the global level of cooperation in a simulation model that assumes that linking costs are high enough.

Riedl and Ule’s experiment (2002) confirms that cooperation is significantly higher when players have the option to reject partnership offers. Experiment subjects use exclusion as

⁸ All dynamic models assume random global search for partners: the link update process.

punishment: they tend to reject links with known defectors much more often than with cooperators and what is more, they do this even when it is costly. The effect from exclusion on cooperation is particularly strong when the exclusion option is cheap (which could be seen as equivalent to high link costs) and when players observe the behavior of all other players (they have global information). Since the threat of exclusion is more credible in such a situation, experiment subjects tend to cooperate more. Yet, in simulations, agents are rarely endowed with such foresight. Nevertheless, the numerical experiment of Hanaki et al. (2007) confirms exclusion as the mechanism that fosters cooperation when linking costs are high. In this case, rather than to discourage defection, the role of exclusion is to stem its spread through imitation.

If players tend to exclude defectors, one can expect that the higher the number of cooperators, the higher the density of the resulting interaction network. In their experiment, Riedl and Ule (2002) find out that network density is indeed positively correlated with cooperation but only when exclusion is cheap (or alternatively, maintaining links is costly).

The successful isolation of defectors can further lead to segregation, that is, the segmentation of the network into components consisting of either only cooperators or only defectors. In Mengel and Fosco's model (2007), segregation occurs when players have some (but not global) information beyond their interaction radius. Here, the localization of information has a twofold effect: on the one hand, it allows defection to spread locally and to destabilize some cooperative components and on the other, it also enables cooperators to find and connect to other cooperators and hence, to exclude defectors from their cooperative clusters. As such complex dynamics are missing in Riedl and Ule's six-person experimental groups (2002), the researchers discover another mechanism that enables segregation. When information is limited to one's interaction partners only and exclusion is cheap, segregation occurs because isolated defectors are not aware that a cooperative clique exists.

Cooperation can be maintained not only through the isolation of defectors but also through their marginalization. The resulting structure is a connected hierarchical network with a large number of cooperators in the center and a few defectors at the fringes. Two available simulation studies obtain this kind of structures. On the one hand, Eguíluz et al. (2005) use a deterministic model in which only newly converted defectors randomly search for new partners. They find out that the hierarchical structure gets accentuated (the degree distribution significantly departs from the Poisson distribution of a random network towards an exponential one) with increasing network fluidity and increasing temptation to defect. In comparison, in Mengel and Fosco's stochastic model with local partner search (2007), connected hierarchical networks are possible only when players choose partners among

“friends of friends” (search radius $r = 2$). Since in this case one’s neighbors are likely to be also related, the global structure exhibits high clustering, yet a short average distance between nodes, a type of network known as a “small world” (Watts and Strogatz, 1998). Although their model differs in a number of assumptions, Eguíluz et al. (2005) also obtain the same result when they specify local search.

The fact that players with different behavior occupy different positions in the hierarchical network implies that they assume different roles. Depending on who imitates whom, Eguíluz et al. (2005) differentiate three roles: leaders (highly connected cooperators who have the highest payoff in their neighborhood), conformists (cooperators who imitate leaders) and exploiters (defectors marginalized to the periphery who are not imitated by anyone). Since one’s payoff in the social game depends on the number of one’s partners, as well as their behavior, the role-differentiation implies inequalities in the distribution of payoffs. The leaders have the highest payoffs because they are involved in a large number of cooperative interactions. The conformists receive less because they cooperate with fewer partners and are possibly also exploited by defectors. Yet, the average payoff of defectors is larger than the average payoff of cooperators. Nevertheless, due to the small number of defectors in stable states, Eguíluz et al. (2005) observe that the payoff distribution closely follows the degree distribution, i.e., it is exponential. Hence, just like hierarchization, inequality increases when the network becomes more fluid and/or defection becomes more tempting.

4.3. Hawk-Dove Game

The Hawk-Dove Game combines the coordination and cooperation problems of the previous two games, as well as their specificities concerning equilibrium selection: like the Coordination Game, it allows for non-trivial analytical solutions but similarly to the Prisoner’s Dilemma, it requires additional assumptions to explain the high levels of cooperation observed empirically.

In the social game of Hawk-Dove, the predicted Nash equilibrium configurations do not generally coincide with the states that provide maximum welfare (in the sense of the maximized sum of individual payoffs). When the game represents a cooperation dilemma ($2c > b + d$), the most efficient state is a complete network with only Doves but, as in the Prisoner’s Dilemma, this state is never a Nash equilibrium. When the game represents a coordination dilemma ($2c < b + d$), the number of Doves in the socially optimal states is a function of the payoffs and the link costs but is always between and including $n/2$ and n (Bramoullé et al., 2004). Generally, the Nash equilibrium predictions are not unique but as

link costs decrease, the interval of possible Hawk-Dove configurations narrows down to the mixed-strategy equilibrium point (see Table 2). The intuition behind this is that when link costs are low, players have the incentive to form a complete network and hence, the link formation process no longer influences behavior. Thus, in contrast to the Coordination Game, where efficiency is certain only for high link costs, in Hawk-Dove, more efficient equilibria are only guaranteed for low link costs.

In contradiction to the analytical predictions, Berninghaus et al. (2008) find out that subjects in experiments exhibit high levels of cooperation. In order to account for this observation, they assume that the players are forward-looking and define a “one-step-ahead” stability concept. The concept generally predicts a higher number of cooperators than in the Nash equilibria and even some socially optimal states if link costs are sufficiently high ($b > k > c$).

With regards to the interaction pattern in stable states, the payoff asymmetry in the Hawk-Dove Game generally leads to incomplete network structures. The theoretical prediction is that network density decreases with an increase in link costs. When costs are low, any kind of interaction is profitable so players form the complete network. When costs are high ($b > k > c$) and one-sided, Hawks can maintain links with Doves but not vice versa. Since links between players employing the same strategy are also not profitable, the resulting network is bipartite. The predicted negative relation between link costs and network density has also been confirmed empirically (Berninghaus et al., 2008).

The particular situation with unilateral link-formation and high link costs also carries implications for inequality. Doves earn more than Hawks but their high payoffs depend on Hawks paying the links: if everybody is a Dove, no connections will be established and everybody’s payoff will be zero. Thus, in this case, the roles reverse and Doves become the exploiters.

Apart from role-differentiation, high link costs can also give rise to complex reciprocity patterns. Berninghaus et al. (2008) observe two particular configurations among their experiment subjects: bilateral sequential sponsoring (taking turns to pay for the communication costs) and circular sponsoring among three or more Doves. Essentially, limited opportunities to earn a positive payoff can instigate people to coordinate on more complex forms of cooperation.

5. Discussion and conclusion

By analyzing existing models, isolating their parameters and then linking them to macro-level outcomes, this paper attempted to answer the question of how the coevolution of networks and cooperation or coordination can be modeled. The analysis revealed that the most commonly studied parameters are the cost of links relative to the payoffs of the underlying game and the frequency with which players change partners rather than action. Analytical predictions, simulation results, and experimental findings all suggest that the outcomes are not always efficient and almost never unique. Nevertheless, certain trends have become apparent.

Firstly, high link costs decrease network density. This is an intuitive result: the more expensive links are, the less links one is willing to maintain. In addition, however, depending on how the link costs relate to the payoffs of the underlying game, individual incentives change and lead to different predictions on welfare at the macro-level. For example, in the Coordination Game, high link costs guarantee efficiency, while in the Hawk-Dove Game, only low link costs guarantee Nash equilibria closer to the socially optimal outcome. The experiments on cooperative games also confirm that the cost of links carries major implications for efficiency: the number of cooperators is significantly higher for high link costs in both Hawk-Dove and the Prisoner's Dilemma (in the latter case, provided that players have global information).

Secondly, "faster" networks lead to more dramatic macro-patterns. In simulation studies, the faster the speed of network update relative to the speed of action update, the higher the level of polarization in segregated networks and the higher the degree of inequality in connectivity and payoffs in connected networks. Faster networks also enhance efficiency in the Prisoner's Dilemma if players have big information radii.⁹

The effect of the dynamics of the link update process, however, is conditional on the assumption that actors do not discriminate in action choice among their partners. When one uses the same action against everybody one is connected with, changing one's behavior has bigger consequences than changing a tie. This is because a link change affects only a single interaction while a behavior change affects all of one's interactions. Thus, since actors tend to exhibit a certain inertia in behavior, when they update their links more often, their action choices accumulate and persist to greater degree and hence, more extreme structures are produced.

⁹ Analogously, in their model of Stag-Hunt with endogenous partner selection, Skyrms and Pemantle (2000) explore two different probabilities for the random selection of a player to change her strategy to the most successful one. They find out that the slower strategic adaptation (i.e. the faster network) leads to coordination on the payoff dominant strategy.

Since the assumption of non-discriminatory action is common to all reviewed studies, I did not discuss it in the section on how to build a coevolution model. However, as it might lead to an overestimation of the effect of network fluidity on macro-level outcomes, it deserves special attention. Before I can conclude with some general remarks and suggestions for further research, I elaborate on the origin of this assumption and possible consequences from relaxing it.

5.1. Assumption of non-discriminatory action choices

Researchers commonly justify the assumption of non-discriminatory action choices as the very condition that enables network effects (e.g. Riedl and Ule, 2002; Bramoullé et al., 2004; Goyal and Vega-Redondo, 2005). A player's action change triggers a reaction by her direct neighbors, which in turn affects the neighbors' neighbors. This allows for cascade dynamics, where a single individual's action choice propagates through the system and radically alters the global behavior. However, individual adjustments in the interaction network could stop such extreme network dynamics and produce more complex self-organization patterns.

The assumption of actors employing the same action towards all their neighbors can be traced back to one of the predecessors of behavior-network coevolution research, namely, the study of cellular automata.¹⁰ The assumption is also conventional in evolutionary game theory, which studies strategy drifts in populations. Originally developed in computability theory and evolutionary biology, respectively, these two research areas focus on actor states or types, rather than on actions or relations. In the social sciences, the assumption of non-discriminatory action choices is justifiable in (anti-)coordination games applied to personal characteristics that are costly to be "customized" per interaction, such as political views or professional specialization. However, non-discriminatory action is a strong behavioral assumption as far as cooperation is concerned: most of us have both profitable and adverse relationships. Allowing players to behave differently with each partner offers not only more socially realistic models, but also new directions for research.

5.2. Discriminatory action choices

Introducing relationship-specific actions to social games shifts the research focus from long-term population evolution to short-term group dynamics. This shift implies new substantive

¹⁰ A cellular automaton is a model of a fixed grid of cells, where each cell has a finite number of states and where the cell's current state depends (following a simple universal rule) on the states of the cell's neighbors in the previous time period.

questions, new equilibrium predictions, and more complex behavioral patterns in experiments.

First, the assumption of discriminatory action choices allows us to re-investigate the question whether positive or negative sanctions facilitate cooperation and coordination. If players can behave differently with different partners, they can punish exploitative actors through costly miscoordination or defection. In comparison, the current models allow only for punishment through exclusion and in order to render it costly, they need to introduce additional payoff conditions (see for example, Riedl and Ule, 2002). Discriminatory action choices enable us to use social games to re-examine not only the effect of costly punishment on cooperation, but also the role of direct and indirect reciprocity. Corten and Cook (2008) provide a first attempt in this direction: they model players who reciprocate the behavior expected from their partners, where the players' expectations are based on own previous experience and information from third parties (i.e. reputation). The possible conflict between direct and indirect reciprocity itself raises additional research questions: How do people behave when someone is "nice" to them but "nasty" to their friends? Do conflicts between direct and indirect reciprocity lead to the emergence of group norms that induce and maintain more efficient outcomes?

Second, the assumption of discriminatory actions could also lead to different equilibrium predictions. Goyal and Janssen's study (1997) on non-exclusive conventions in static networks offers some preliminary evidence. The authors model actors on a circle who play a Coordination Game, in which they can adopt both actions for a certain "action flexibility" cost. The model predicts co-existing conventions for a certain range of flexibility costs. In comparison, without action flexibility, no co-existence is possible among players on a circle who interact locally (e.g. Berninghaus and Schwalbe, 1996).

Last but not least, allowing players to relate differently to their partners could also allow us to observe more complex reciprocity patterns in experiments. Laboratory studies of individuals simultaneously playing multiple independent bilateral games show that probabilistic behavior is common in experiments (Hauk and Nagel, 2001; Bolton et al., 1998). Most experiment participants act differently against different partners even in the first round when everybody is the same *ex ante*. Hence, Hawk and Nagel (2001) hint that the multiple-game technique may be useful for studying games with mixed equilibria (e.g. Hawk-Dove) in an experimental setting. I would take this idea one step further and suggest that experimentation through probabilistic choices could also allow individuals to coordinate on more complex time and spatial patterns of reciprocity.

5.3. Major gaps and directions for research

Overall, the systematic overview of existing models of the coevolution of behavior and networks revealed few overlaps and multiple gaps in model development. These often frustrate attempts to draw universal conclusions about mechanisms without the specificities of the models. Yet, the gaps also indicate numerous areas for future theoretical and empirical exploration. I would like to conclude by recommending two major points that future research should address.

In the first place, models should relate to applications more explicitly. Current coevolution research is often driven by analytical tractability rather than application-relevant or empirically valid assumptions. In particular, I argued that the assumption of actors employing the same action against all of their interaction partners corresponds neither to the nature of social relations, nor to the short-run group dynamics experiments test. This brings us to the second point: future coevolution research in the social sciences should provide more experimental evidence. The three analyzed experimental studies demonstrate that Nash solutions are poor predictors of behavior observed in the laboratory. Empirical tests serve to identify problems with current theories and thus, stimulate the development of more sophisticated models with more predictive power and more universal applications.

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